

B.Sc. Part I
Paper I

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R Theory of Relativity

Problem:- The life of μ -mesons is 2.2×10^{-6} sec. and their speed is $0.998c$; so that they cover only a distance of $0.998c \times 2.2 \times 10^{-6}$ or 658.6 metres in their entire life time and yet they are found in profusion at sea level, i.e., at a depth of 10 km from the upper atmosphere where they are produced. How may this be explained on the basis of (i) Lorentz-Fitzgerald contraction. (ii) time dilation.

Sol:- (i) Here 2.2×10^{-6} sec is the proper mean life time and the distance covered by them during this time = 658.6 m.

The distance traversed by them as observed from earth

$$L = \frac{L_0}{\sqrt{1 - \frac{v^2}{c^2}}} = \frac{658.6}{\sqrt{1 - \left(\frac{0.998c}{c}\right)^2}}$$

$$= \frac{658.6}{0.064} = 10290 \text{ m} = 10.29 \text{ km}$$

Thus μ -mesons can cover a distance of 10 km or more despite their short life time. This explains their presence at sea level on the basis of length contraction.

(ii)

the observed life of μ -mesons

$$\Delta t' = \frac{\Delta t}{\sqrt{1 - \frac{v^2}{c^2}}}$$

$$= \frac{2.2 \times 10^{-6}}{\sqrt{1 - \left(\frac{0.998c}{c}\right)^2}}$$

$$= \frac{2.2 \times 10^{-6}}{0.064} \text{ sec}$$

$$= 34.6 \times 10^{-6} \text{ sec}$$

i.e., distance covered in this time

$$= v \Delta t' = 0.998c \times 34.36 \times 10^{-6}$$

$$= 10290 \text{ m}$$

$$= 10.29 \text{ km}$$

This explains their presence at sea level on the basis of time dilation.

4) Relativity of time; Proper time :-

The time recorded by a clock moving with a given system is called Proper time for that system.

Let there be two systems S and S' , S' moving with a velocity v relative to the system S along (+)ve direction of x -axis. Imagine the system S' to be an aeroplane moving with velocity v and an observer is in the aeroplane and the other system S , i.e., at rest. Let the observer at rest measure the time t for the journey of the observer in the aeroplane and if t' is the corresponding time recorded by the observer in the plane, then according to Lorentz transformation equations,

$$t' = \frac{t - \frac{vx}{c^2}}{\sqrt{(1-\beta^2)}}$$

Since t is the time recorded by an observer at rest and v is the velocity of the plane, therefore $x = vt$ (\because distance = velocity \times time)

Putting this value in above equation, we get

$$\begin{aligned} t' &= \frac{t - \frac{v \cdot vt}{c^2}}{\sqrt{(1-\beta^2)}} = \frac{t(1 - v^2/c^2)}{\sqrt{(1-\beta^2)}} \\ &= \frac{t(1-\beta^2)}{\sqrt{(1-\beta^2)}} = t\sqrt{(1-\beta^2)} \end{aligned}$$

or

$$t' = t\sqrt{\left(1 - \frac{v^2}{c^2}\right)}$$

So that $t > t'$

Thus time recorded by the two clocks in two systems S and S' for the same journey are different i.e., two clocks in the two systems S and S' run at different rates. We thus conclude that there are two proper times, one for observer in system S and other for observer in S' . We have the time recorded in system S to be greater than that system S' . By the principle of relativity we have observer in S' to be at rest relative to the plane and the observer in S has a velocity v relative to the plane. Conclusively "A moving clock runs more slowly than a stationary one." Also time is not absolute as considered in Galilean transformations; but relative only.